Parallel Strategies Selection

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Abstract. We consider the problem of selecting the best variable-value strategy for solving a given problem in constraint programming. We show that the recent Embarrassingly Parallel Search method (EPS) can be used for this purpose. EPS proposes to solve a problem by decomposing it in many subproblems and to give them on-demand to workers which run in parallel. Our method uses a sample of these subproblems for comparing strategies in order to select the most promising one to be used for solving the remaining subproblems. Each subproblem of the sample is solved with all the candidate strategies in parallel using a timeout that is twice the time of the best one. The selection of the strategy is then based on the Wilcoxon signed rank test. This test is able to deal with censored data caused by timeouts and makes no assumption on the solving time distribution. The experiments we performed on a set of classical benchmarks for satisfaction and optimization problems show that our method selects most of the time the best strategy. Our method also outperforms the portfolio approach consisting of running some strategies in parallel and is competitive with the multi armed bandit framework.

1 Introduction

Many generic variable-value strategies have been imagined [23,19,5,8,27,17]. Those are especially useful in the absence of specific knowledge on the problem to solve. They either try to apply generic principles like the first fail principle (i.e. try to fail as quickly as possible) [13] or try to detect underlined relations between variables and constraints. In the first case, we have strategies like min-domain which selects the variable having the minimum domain size, max-constrained which prefers variables involved in a lot of constraints, or min-regret which selects the variable which may lead to the largest increase in the cost if it is not selected. The latter case is mainly formed by the impact based strategy [23], weighted degree strategy [5] and the activity based strategy [19]. More recently strategies attempting to prioritize variables according to the past failures or conflicting decision have also been designed [8,27,17].

However, selecting *a priori* the best variable-value strategy is not an easy task. Indeed no strategy dominates the other ones in general and it is difficult to identify the types of problems for which a strategy performs well. Any variable-value strategy can give good results for a problem and really bad results for some others. It is not rare to

see ratio of performance for a pair of strategy going up from 1 to 20 (and even more sometimes) according to the problems which are solved.

Unfortunately, there is almost no way to compare the performance of variable-value strategies on a problem without solving it. Since strategies explore the search space in different ways and since their pruning performances are not regular it is difficult to compare their behavior before the end of the resolution.

Selecting the right strategy is a challenging decision impacting drastically the solving time.

Our problem can also be seen as the automatic selection of the most efficient algorithm among a predefined set of algorithms, for solving a given problem [26,15,16]. Usually two types of approaches are considered [7]. Either we try to determine statically, that is *a priori*, which is the best algorithm or we dynamically compute the best algorithm to use for each step of the problem solving. Both cases use a set of instances of the problem from which they learn different criteria that will be used to take a decision.

We propose an original approach which is not based on machine learning but on the statistical estimation of the best algorithm. Our approach does not require to deal with a set of instances and use some sampling technique that are usually more accurate. It exploits the decomposition proposed by the embarrassingly parallel search (EPS) method recently developed [24,25].

EPS proposes to solve a problem by decomposing it into a large number of subproblems consistent with the propagation (i.e., there is no immediate failure triggered by the initial propagation of a subproblem). We propose to use a part of these subproblems for comparing the strategies in order to select the most promising one for solving the whole problem. Instead of comparing the strategies after solving the whole problem, we compare the strategies for each subproblem of the sample. We measure the solving time for each subproblem and each strategy and we eliminate the strategies that are statistically proved to be less efficient by a Wilcoxon signed rank test. At the end, either only one strategy remains or a set of non distinguishable strategies. In this latter case we select the one having the smallest mean.

Since for each subproblem the solving times for the strategies may strongly vary, it is necessary to add a timeout mechanism to control the time spent in the strategy selection and to stop some computations after a given amount of time. From a statistical point of view, this means that we may have censored data. By defining appropriately these timeouts, we show that the results of the Wilcoxon signed rank test remains valid if timeouts were not considered. Solving each subproblem with each strategy in parallel allows us to define relative timeouts: we stop a strategy when it requires more than twice the solving time of the best strategy.

It is important to note that our method does not require to know the distribution of the solving times (we made some experiments showing that the distributions vary according to the problems or to the strategy, and there are no general guidelines).

Our method can be distinguished from the machine learning approaches in two ways:

– The relation between the data from which we take our decision and the instance to solve is stronger in our case because we consider subproblems of the instance and not some other instances.
– We do not try to learn any criteria and we do no try to estimate solving times. We instead aim at selecting the most promising strategy for the given instance only. Our results are statistically validated.

The paper is organized as follows. First we show the principles of our method on an example. Then, we recall some preliminaries. Next, we detail the different steps of our approach. We present some related work and some experiments on a set of benchmarks, for which we compare our results with classical portfolio and a multi-armed bandit method. At last we conclude.

2 Selection Principles

We present the principles of our method on a didactic example obtained from the all-interval series, a common benchmark.

Our method proceeds by elimination of strategies until there is only one remaining.

We consider 4 strategies (S_1, S_2, S_3, S_4). The initial problem has been decomposed into 300 subproblems from which we randomly select only 10 subproblems for the sake of clarity.

We could consider each subproblem in turn and run all the strategies on it in parallel. The drawback of this approach is that the running times are not regular and that some strategies may perform poorly for some subproblems compared to other strategies. For instance, here are the runtimes (in milliseconds) for each subproblem:

subproblem	S_1	S_2	S_3	S_4
1	62	408	80	150
2	90	1 134	92	154
3	155	1 904	158	233
4	231	1 451	250	407
5	198	1 580	197	422
6	146	803	170	144
7	62	611	54	115
8	63	389	111	86
9	167	560	163	670
10	83	736	120	232
Σ	1 257	9 576	1 395	2 613

With this approach the total time for selecting the best strategy is $1257 + 9576 + 1395 + 2613 = 14841$, that is more than 10 times the best runtime. Since there are 300 subproblems to solve and since we selected 10, then we can expect a total solving time around 30 times the runtime of the best strategy for our 10 subproblems that is $1.26 \times 30 = 37.8s$[1]. This means that the time allocated to the strategy selection,

[1] We do not claim that this computation is accurate. We present it only for understanding the intuitive idea.

named selection time, may require more than 40% of the solving time. In practice running all the strategies on each subproblem in the sample might take up to 90% of the solving time that would be taken by the best strategy to solve all the subproblems. Our objective is to keep the overhead induced by the selection strategy minimal. Therefore some timeouts are be introduced with respect to the time of the best strategy on each subproblem. Timeouts may cause censored measures that must be carefully treated by statistical methods.

We propose to deal with censored data and proceed by steps.

1. For each subproblem we compare the strategies, but we introduce a timeout limit for each computation corresponding to 2 times the runtime obtained by the best strategy.
2. We select the strategy having the smallest total time (timeouts are counted as their values). If this strategy was stopped by a timeout for some subproblems we run it again on these subproblems without timeouts. We repeat this step until the strategy having the smallest total time without timeout, which we denote s_b, has been selected.
3. We compare all the strategies against s_b by using the Wilcoxon signed rank test. All strategies significantly slower than s_b are eliminated. If s_b is rejected by the Wilcoxon test against s_x (in theory this can happens even if s_b has a better mean) then s_b is eliminated and replaced by s_x. Note that this latter case never happens in the 10,000s of tests we made.
4. Eventually, if some strategies cannot be distinguished by the Wilcoxon signed rank test then we select the strategy performing the best on the sample.

Note that in any case we have a strong statistical support of our choice.

With timeouts corresponding to twice the runtime of the best strategy for each subproblem we obtain the following table:

#	timeout	S_1	S_2	S_3	S_4
1	$2 \times 62 = 124$	62	TO	80	TO
2	$2 \times 90 = 180$	90	TO	92	154
3	$2 \times 155 = 310$	155	TO	158	233
4	$2 \times 231 = 462$	231	TO	250	407
5	$2 \times 197 = 394$	198	TO	197	TO
6	$2 \times 144 = 288$	146	TO	170	144
7	$2 \times 54 = 108$	62	TO	54	TO
8	$2 \times 63 = 126$	63	TO	111	86
9	$2 \times 163 = 326$	167	TO	163	TO
10	$2 \times 83 = 166$	83	TO	120	TO
Σ		1 257	2 484	1 395	2 142

It is important to remark that the best strategy for the whole problem is not the best one for each subproblem. In practice it happens frequently that the best strategy has some timeouts.

The Wilcoxon signed rank test considers the difference in response within pairs. Then it ranks the absolute values of these differences. The sum W^+ of the ranks for the

positive difference is the *Wilcoxon signed rank statistic* and has mean $\mu_{W^+} = \frac{n(n+1)}{4}$. The Wilcoxon signed rank test rejects the hypothesis that there is no systematic differences within pairs when the rank sum W^+ is far from its mean.

Suppose we want to compare the strategies S_1 and S_3. For each subproblem we compute the difference $time(S_1) - time(S_3)$. Then, we rank the absolute values of these differences and we add a sign in front of these ranks corresponding of the signs of the differences. For instance, for the first subproblem we have $time(S_1) - time(S_3) = 62 - 80 = -16$, 16 is the 6^{th} values so its rank is 6. The sign rank is -6 because the difference is negative. Then, we compute W^+, the sum of the positive ranks. The following table shows that we have $W^+ = 1 + 5 + 4 = 10$.

sub problem	S_1	S_3	$S_1 - S_3$	signed rank
1	62	80	-18	-6
2	90	92	-2	-2
3	155	158	-3	-3
4	231	250	-19	-7
5	198	197	1	+1
6	146	170	-24	-8
7	62	54	8	+5
8	63	111	-48	-10
9	167	163	4	+4
10	83	120	-37	-9

We consider a one-tailed test ($S_3 = S_1$ or $S_3 > S_1$) with a significance level of 0.05.

The critical value of W for $N = 10$ at $p \leq 0.05$ is 10. Therefore the result is significant and we can conclude that S_1 is better than S_3. So, we can eliminate S_3.

We repeat this process between S_1 and the other strategies. We will prove that we can perform the calculations by using the timeouts values if these values are defined by any value greater than twice the maximum positive difference because in this case the positive ranks will not change for any value greater than this timeout. For instance, when we compare S_1 and S_4 there is only one positive difference equal to 2 (for subproblem 6, we have $146 - 144 = 2$), so for each subproblem j we can set the timeout to any value v such that $v > time(S_1, j)$ and $|time(S_1, j) - v| > 2$, because this will not impact the rank of value 2 and so the value of W^+.

If we apply this process for our example, the comparison against S_1 will eliminate all the other strategies.

In conclusion, S_1 is selected. This leads to a resolution time of about 39.4s.

3 Background

3.1 Statistics

These definitions are due to Moore et al. [21].

Simple random samples. A *simple random sample (SRS)* of size n consists of n individuals from the population chosen in such a way that every set of n individuals has an equal chance to be the sample actually selected. We select an SRS by labeling all the individuals in the population and selecting randomly a sample of the desired size. Notice that an SRS not only gives each individual an equal chance to be chosen (thus avoiding bias in the choice) but gives every possible sample an equal chance to be chosen.

Wilcoxon Signed Rank Test for Matched Pairs. Our data do not follow a Normal distribution and timeouts are introduced leading to right censored data. Common method like t-test can thus not be used and non nonparametric tests have to be considered instead for comparing strategies. We use the Wilcoxon Signed Rank Test. Bootstrap methods and permutation tests based on the idea of applying the method many times would be too time-consuming for our purpose.

Since we aim at comparing the performance of two algorithms we consider a *matched pairs* design, which compares just two observations. The idea is that matched subjects are more similar than unmatched subjects, so comparing responses within a number of pairs is more efficient than comparing the responses of groups of randomly assigned subjects. Matched pairs data are analyzed by taking the difference within the matched pairs to produce a single sample. The one sample statistic is applied on this difference data in order to compare the matched pairs data.

The *Wilcoxon signed rank test* (WSR test) for matched pairs is defined as follows. Draw an SRS of size n from a population for a matched pairs study and take the difference in responses within pairs. Rank the absolute values of these differences. The sum W^+ of the ranks for the positive difference is the *Wilcoxon signed rank statistic*. If the distribution of the responses is not affected by the different treatments within pairs, then W^+ has mean $\mu_{W^+} = \frac{n(n+1)}{4}$ and standard deviation $\sigma_{W^+} = \sqrt{\frac{n(n+1)(2n+1)}{24}}$. Difference of zero are discarded before ranking. Ties among the absolute differences are handled by assigning average ranks.

The WSR test rejects the hypothesis that there is no systematic difference within pairs when the rank sum W^+ is far from its mean.

P-values (i.e., the probability computed assuming that null hypothesis is true, that the test statistic will take a value at least as extreme as that actually observed) for the signed rank test are based on the sampling distribution of W^+ when the null hypothesis is true. P-values can be computed from the exact distribution (from software or tables) or obtained from a Normal approximation with continuity correction.

3.2 Embarrassingly Parallel Search [25]

The idea of the Embarassingly Parallel Search (EPS) is to decompose statically the initial problem into a huge number of subproblems that are consistent with propagation (i.e., running the propagation mechanism on them does not detect any inconsistency). These subproblems are added to a queue which is managed by a master. Then, each idle worker takes a subproblem from the queue and solves it. The process is repeated until all the subproblems have been solved.

The decomposition is made by selecting a set V of k variables and then by searching all instantiations of V that are consistent with propagation. There is no specific variable-value strategy used to find these instantiations. The number of generated subproblems depends on the size of V which is determine by successive computations.

The assignment of the subproblems to workers is dynamic and there is no communication between the workers. EPS is based on the idea that if there is a large number of subproblems to solve then the resolution times of the workers will be balanced even if the resolution times of the subproblems are not. In other words, load balancing is automatically obtained in a statistical sense. Interestingly, some experiments [24] have shown that the number of subproblems does not depend on the initial problem but rather on the number of workers. Moreover, they have shown that a good decomposition has to generate more than 30 subproblems per worker.

4 Method

4.1 Simple random sample

We use EPS to decompose the initial problem into a huge set of subproblems. Thus the population is the set of these subproblems. The SRS is built by selecting randomly k subproblems from the set of subproblems. The sample is limited to 1% of the subproblems to avoid spending too much time for the strategy selection. If $k = 30$ subproblems seems to be the minimum number of subproblem to consider, then we need to have at least $3,000$ subproblems.

4.2 Comparison of Strategies

Strategies are compared by using the WSR test on the SRS previously defined. For each subproblem of the SRS we run the strategies in parallel and we stop the slowest ones when they require twice the time of the best strategy. Then, we select the strategy having the smallest sum of solving times for all the subproblems of the SRS. If this strategy was stopped by a timeout for some subproblems we run it again on these subproblems without timeout. We repeat this step until the strategy, denoted by S_b, having the smallest total time without timeout has been selected. Next, we compare all the strategies against S_b by using the WSR test performed on some modified data. All strategies significantly slower than S_b are eliminated. If at a moment, the strategy S_b is rejected by the Wilcoxon test against another strategy S_x, then timeouts are removed for S_x and we use a t-test for deciding whether S_x should become the best strategy. In this latter case we simply replace S_b by S_x.

In any case, we have a strong statistical support of our selection.

Our hypotheses are

H_0: there is no difference between data of both Strategies.

H_a: scores are systematically higher for the second Strategy.

In order to make sure that the result of the test remains valid when exact solving times are considered instead of timeout values, we proceed as follows. Suppose we compare S_b and S_i. Let us show that if we set for each subproblem j the timeout to a

value $to(j) > d_{bi}^{\max} + time(S_b, j)$ where $d_{bi}^{\max}$ the largest positive value of $time(S_b) - time(S_i)$ for all the subproblems of the SRS then the test is valid if exact solving times are considered instead of timeouts.

Property 1 *Let $d_{bi}^{\max}$ the largest positive value of $time(S_b, j) - time(S_i, j)$, and $rank(d_{bi}^{\max})$ be its rank in the WSR test of that value. Then, $rank(d_{bi}^{\max})$ is the greatest value of W^+ and for any value v such that $rank(v) > rank(d_{bi}^{\max})$ we have $time(S_b, j) - time(S_i, j) < 0$ and $v > d_{bi}^{\max}$.*

proof: By definition of the ranks and since $d_{bi}^{\max}$ is the largest positive value of $time(S_b, j) - time(S_i, j)$ then it has the largest rank in W^+, thus any value having an absolute value greater than $d_{bi}^{\max}$ is negative and has a greater rank $\odot$

Property 2 *Suppose that for any subproblem j the timeout for j is set for S_i to a value $to(j) > d_{bi}^{\max} + time(S_b, j)$ and let W^+ be the sum computed with these timeouts. Then, for any value of timeout greater than $to(j)$ the value of W^+ remains unchanged.*

proof: If the timeout is set to $to(j) > d_{bi}^{\max} + time(S_b, j)$ then for any j reaching the timeout $|time(S_b) - time(S_i)| > d_{bi}^{\max}$. From Property 1 the increase of $to(j)$ will not change the rank of the elements of W^+ so the property holds $\odot$.

So, for each subproblem j such that S_i has been stopped by a limit which is less than $d_{bi}^{\max} + time(S_b, j)$, we solve again this subproblem with S_i with the time limit defined by $d_{bi}^{\max} + time(S_b, j) + 1$. Therefore, our deduction are statistically valid.

At the end, it is possible that we cannot deduce that some strategies are statistically different. However, this means that they should lead to equivalent solving time for the whole problem, so we can select any of them. In this case, we select the strategy performing the best on the sample.

If we compare s strategies with an initial timeout fixed to twice the time of the best strategy and if $tmax(S_b)$ denotes the largest solving time of a subproblem of the sample by the best strategy S_b, then the sum of the solving times for all the strategies for each problem in the sample is bounded by $s \times tmax(S_b)$.

Significance level of the results The significance level of the method is bounded by the product of the confidence intervals of each comparison. This means that for k comparisons, each with a confidence interval of 99%, the overall result has a confidence interval of 0.99^{k-1}. Fortunately we have only few strategies. For instance for 7 strategies, this leads to a confidence level of $0.99^6 = 94\%$. This is quite acceptable.

Optimization Problems In optimizations problems, an optimal value of an objective function has to be found, thus bounds on this function are important. For each subproblem, all strategies have the same bound. When a bound is found for subproblem i, it is used for all subproblems considered after i for all strategies

5 Related work

There has been a significant amount of work on automatically selecting or adapting the search strategy. Some successes have been obtained by running some algorithms in parallel in CP [10] and in SAT [12]. Offline and online machine learning based methods are popular. Offline methods select automatically the strategy among a set of available strategies. They perform a learning phase on a training set of instances. They haven been initially proposed for SAT [28] and then for CSPs [22]. Hamadi [11] proposed two methods: continuous search which aims at finding the best strategy for solving a given problem and autonomous search which aims at finding the best strategy in general. These methods are based on machine learning techniques. On the other hand, online methods have been considered. Epstein et al. [6] proposed Adaptive Constraint Engine (ACE), a method which gathers the decision made by several strategies and proceed to a vote in order to decide which one will be applied for the next decision. Gagliolo and Schmidhuber [7] allocate times to each algorithms by using a multi-armed bandit algorithm whose decisions is based on the previous computations. Arbelaez et al. [1] apply Support Vector Machines to the problem of automatically adapting the search strategy of a CP solver in order to more efficiently solve a given instance. Loth et al. [18] define the best strategy during the search by using a multi-armed bandit approaches combined with Monte Carlo Tree Search. Racing algorithms using a non-parametric test based on ranking to successively discard unpromising configurations, like F-Race [3], have also been proposed. However, F-Race does not deal with censored data: it successively executes the algorithm until its completion on new sampled problems. There is no parallel execution and no time-out that is central to our approach.

For a good introduction to Algorithm selection we encourage the reader to refer to [7] and [16].

6 Experiments

All the experiments have been run in parallel on a parallel machine. The scaling of the EPS method does not depend on the problem solved, so it is the same for all the variable-value strategies. Therefore, for each strategy we have used the sum of the time spent on each core allocated to this strategy as a measure of the time required by the strategy. The best of these times correspond to the value we want to minimize, thus our experiments are based on these times.

Machines All the experiments have been made on a Dell machine having four E7-4870 Intel processors, each having 10 cores with 256 GB of memory and running under Scientific Linux.

Solver We implemented our method on the top of Gecode 4.2 (http://www.gecode.org/).

Considered strategies After some experiments we selected 7 candidate strategies. Each strategy is dynamic:

- FF implements the first fail principle by selecting the variable with the minimum domain size [13];
- Act selects the variable with the maximum of activity [2] [19];
- Wdegm selects the variable with the maximum weighted degree [3] [4];
- WdegM same as above excepted that the value is selected differently;
- MRegret selects the variable for which the difference between the largest and second-largest value still in the domain is maximum [9].
- MostC selects the most constrained variable.
- D/Wdeg selects the variable for which the ratio of the size of its domain by its weighted degree is minimum [4,5].

After selecting the variable, all strategies but Wdegm, assign to it the minimum value of its domain. WdegM assigns to it the maximum value of its domain. We did not consider impact based strategy [23] because this strategy is not implemented in Gecode.

Benchmarks instances We present the most representative results that we obtained (results for other problems are equivalent).

Problems come from the CSPLib, the minizinc challenge [20] or the Hakank's constraint programming blog [14].

For satisfaction problems we search for all solutions and we consider the following problems: all-I: All intervall series 14; Costa: Costa Array 13; Filo: Filomino 13; Lams 9; Qgrp: Quasi group 7; Msplt: Market split s5-08; Sched: sport scheduling 12; Tank: tank attack puzzle 7; Gol: Golomb 12; Perm: Permutation 12.

For optimization problems, we search for the optimal solution and we prove the optimality. Results are given for the following problems: Crew; Dud: dudney thea; Java: java routing trip 6-3; mario; mario medium 3; Fback: minimum feed back; matching problem Money: money change 27; War: War Peace 8; Sugi: Sugiyama 7 7;

Sampling The initial problem is decomposed into 16,635 subproblems from which we randomly select 100 subproblems.

6.1 Main results

PSS denotes the Parallel Strategies Selection that we propose.

Times are expressed in minutes and correspond to the sum of the times spent by all the cores. Bold times indicate the best strategy for the considered problem.

[2] Roughly the activity is defined by the number of times the variables has been introduced in the propagation queue. The activity is increased at most by one for each decision.

[3] The weighted degree of a variable is defined by a counter associated with it. Each time a constraint fails, the counter of each variable involved in the constraint is increased by one.

	FF	Act	Wdegm	WdegM	MRegret	MostC	D/Wdeg	PSS
All-I	26.3	210	55.1	54.4	31.6	26.1	**0.8**	0.9
Costa	**46.2**	365	78.2	153	213	41.7	96.9	49.2
Filo	427	160	**12.0**	78.2	335	654	23.5	12.4
Lams	**58.6**	802	416.3	319.2	49.9	48.7	1301	62.0
Qgrp	36.7	41.0	367	877	4.6	3.3	**2.8**	3.0
Msplt	525	1035	616	620	526	**492**	703	515
Sprt	55.8	265	124	116	73.0	36.6	**14.9**	15.4
Tank	29.6	1091	27K	47K	40.6	13K	**3.8**	4.1
Gol	341	295	543	455	183	334	**168**	176
Perm	234	177	159	201	121	331	**27.3**	28.1

In terms of ratio with respect to the best time (i.e., each time is divided by the best time), we obtain the following table which clearly shows the strong disparities between strategies, and that the performance of PSS is close to the one of the best strategy for each problem. We use the following notation: $\overline{x}$ is the mean and geo $\overline{x}$ is the geometric mean.

	FF	Act	Wdegm	WdegM	MRegret	MostC	D/Wdeg	PSS
All-I	32	254	67	66	38.4	31.7	1	1.06
Costa	1.1	8.8	1.9	3.7	5.1	1.0	2.3	1.06
Filo	35	1.0	13	6.5	27	54	1.96	1.04
Lams	1.2	16.5	8.6	6.6	1.0	1.0	26.8	1.06
Qgrp	13.1	14.7	131	314	1.6	1.2	1.0	1.06
Msplt	1.1	2.1	1.3	1.3	1.1	1.0	1.43	1.04
Sprt	3.7	17.8	8.4	7.8	4.9	2.5	1.0	1.03
Tank	7.9	291	7408	12625	10.8	3576	1.0	1.07
Gol	2.0	1.8	3.2	2.7	1.1	2.0	1.0	1.05
Perm	8.6	6.5	5.8	7.4	4.4	12.1	1.0	1.03
geo $\overline{x}$	5.1	12.1	17.6	19.6	4.4	7.3	1.7	1.05
$\overline{x}$	10.6	61.5	765	1304	9.7	368.3	3.8	1.05

For optimization problems we obtain the following results:

	FF	Act	Wdegm	WdegM	MRegret	MostC	D/Wdeg	PSS
Crew	64	258	85	91	68	**58**	74	61
Dud	**15**	34	40	32	37	17	16.1	16.3
Java	24	35	41	35	**21**	24	108	22.7
Mario	**4.2**	45.9	18.7	9.8	7.4	5.8	5.9	4.6
Fback	**126**	281	379	436	128	131	127	133
Money	**0.6**	0.9	0.9	0.6	0.8	0.6	0.6	0.6
War	185	259	232	250	211	**54**	176	56.4
Sugi	504	154	113	111	381	504	**29.1**	30.1

We can also express them in term of ratios w.r.t. the best time in order to see the relative differences between strategies:

	FF	Act	Wdegm	WdegM	MRegret	MostC	D/Wdeg	PSS
Crew	1.10	4.44	1.46	1.57	1.17	1.00	1.21	1.05
Dud	1.00	2.23	2.60	2.06	2.39	1.12	1.07	1.07
Java	1.12	1.62	1.91	1.64	1.00	1.10	5.14	1.06
Mario	1.0	10.9	4.43	2.33	1.75	1.37	1.40	1.09
Fback	1.00	2.22	3.00	3.45	1.02	1.04	1.01	1.05
money	1.00	1.57	1.57	1.09	1.35	1.12	1.05	1.05
War	3.45	4.82	4.32	4.65	3.92	1.00	3.26	1.05
Sugi	17.3	5.30	3.90	3.80	13.0	17.3	1.00	1.05
geo $\overline{x}$	1.71	3.35	2.67	2.32	2.01	1.56	1.54	1.06
$\overline{x}$	3.38	4.14	2.90	2.57	3.21	3.13	1.88	1.06

Once again our method gives good results. Note that for all problems the Wilcoxon signed rank test was able to eliminate all strategies against the best one.

Next we give some results for the search of the first solution. The chance plays a role in this case. We consider only problems having few solutions since for problems with many solutions the first one is found during the sampling. Times are in minutes and the last column contains the number of subproblems considered before finding one with a solution (recall that the number of subproblems is 16,635). The subproblems are considered as generated by the decomposition. As can be observed the results are surprisingly good with a very limited footprint with respect to the best strategy.

	PSS	Best Strategy	ratio	#firstSol
Filo	5.56	5.45	1.02	5,283
Msplt	26.8	201	1.33	678
Tank	1.94	1.24	1.57	39
Gol	127	125	1.014	12,400

We report next the mean of 250 experiments obtained by randomly selecting the subproblems. The results obtained by PSS are close to the results of the best strategy:

	PSS	Best Strategy	ratio
Filo	7.09	6.19	1.14
Msplt	172	164	1.05
Tank	2.84	2.66	1.07
Gol	101	95	1.07

6.2 Comparison with a Sequential Approach

We compare the results obtained with PSS against the sequential time of the best strategy (Seq+Best) used for each problem. We give wall clock times in minutes.

	All-I	Costa	Filo	Lams	Qgrp	Msplt	Sprt	Tank	Gol	Perm
PSS	0.024	1.29	0.33	1.63	0.08	13.52	0.40	0.11	4.62	0.74
Seq+Best	1.6	34.6	5.95	38.8	2.1	515	9.8	2.9	135	21
Ratio	28.4	26.8	18.3	23.8	26.5	38.1	24.2	26.7	29.1	28.6

6.3 Comparison with Multi-armed Bandit (MAB) Approach

The Multi-Armed Bandit selector is based on a model defined on a set of k arms, one for each strategy, and a set of rewards $R_i(j)$, where $R_i(j)$ is the reward delivered when an arm i has been chosen at time j. A reward reflects the performance of choosing that arm. The idea is to select for each subproblem a strategy (i.e., an arm) and then to solve the subproblem with this strategy. This will give us a reward inversely related to the solving time. The next selection is based on the sequence of the previous trials. We propose to use the UCB1 policy defined in [2], which selects the arm i that maximizes $p(i) = \overline{R}_i + \sqrt{\frac{2lnm}{m_i}}$, where m is the current number of selection, m_i the number of times i has been selected and $\overline{R}_i$ is the mean of the past rewards of the i arm. This policy prefers the most rewarded strategy but also biases the selection toward less frequently selected strategies (this bias factor increases along the iterations). The main difficulty is the definition of the reward function. We adapt the one of Gagliolo and Schmidhuber [7] which is designed for resource allocation and defined by: $\frac{\ln(t_{\max})-\ln(t_i)}{\ln(t_{\max})-\ln(t_{\min})}$, where $t_{\max}$ and $t_{\min}$ are respectively the maximum and minimum solving time and t_i is the time for solving problem i. Experimentally, we obtained the best results by defining $t_{\max} = 10\mu$ and $t_{\min} = \mu/10$ where μ is the mean of the solving times. With such values we accept some variations and degenerate cases (i.e., very bad solving times) will give only negative rewards. We denote by MAB this method. Here is the comparison with PSS:

	time		ratio w.r.t. best	
	PSS	MAB	PSS	MAB
All-I	0.9	2.0	1.06	1.14
Costa	49.2	65.4	1.06	1.41
Filo	12.4	36.8	1.04	3.08
Lams	62.0	102	1.06	1.73
Qgrp	3.0	7.8	1.06	2.77
Mspl	515	548	1.04	1.11
Sprt	15.4	19.8	1.03	1.32
Tank	4.1	12.0	1.07	3.13
Gol	176	243	1.05	1.45
Perm	28.1	31.4	1.03	1.15
geo $\overline{x}$			1.05	1.68
$\overline{x}$			1.05	1.83

The results obtained with PSS are better than with MAB. In addition PSS is more robust. These experiments show that applying the reasoning on subproblems coming from the instance to solve is certainly a good idea.

6.4 Comparison with Portfolio

PSS needs 1172 minutes for solving all the problems. The Portfolio-x4 method runs in parallel the four best strategies. It requires 3959 minutes which is not competitive with our method.

We also tried to combine our approach with a portfolio approach. PSS-pfolio2 is the PSS method for which we run in parallel the two best estimated strategies when the difference between them is small. The following results show that it is never interesting to run some strategies in parallel.

	All-I	Costa	Lams	Qgrp	Msplt	Perm
PSS	0.9	49.2	62.0	3.0	515	28.1
PSS-pfolio2	1.6	94.0	114	5.4	917	37.9

6.5 Timeout, Sample size and Simple impact

The timeout (TO) may have a huge impact on the selection time as shown by the following table, where "without TO" means that we do not stop any strategy when solving a subproblem.

	with TO	without TO
All-I	0.1	4.9
Costa	3.0	16.1
Filo	0.4	17.5
Lams	3.4	9.6
Qgrp	0.2	2.9
Msplt	22.9	82.8
Sprt	0.5	7.8
Tank	0.3	136
Gol	7.9	38.0
Perm	0.8	5.5

We also performed some experiments with a sample size equals to 30 instead of 100. We do not observe any difference for the selected strategy. The best strategy is selected for all problems.

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8 Conclusion

The Embarrassingly Parallel Search method solves a problem by decomposing it into subproblems. In order to select the best variable-value strategy to solve a problem, we propose to use a part of these subproblems and compare some strategies on them. Then, we select the most promising one by using the Wilcoxon signed rank test. This method, PSS, is simple and does not require a lot of computations. It can easily be used in practice because the time allocated to the strategy selection is under control. Some

comparisons with other portfolio approaches show the advantage of our method. We also give a model based on the Multi-armed Bandit algorithm which gives interesting results although inferior and less robust than those of PSS. Finally, it appears that it is better to select only one variable-value strategy than running several in parallel, even if we make some mistakes sometimes.

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